**MODMUL - Modular Multiplication -** [**https://www.spoj.com/problems/MODMUL/**](https://www.spoj.com/problems/MODMUL/)

**🧠 Problem Understanding**

You're given two **non-negative 64-bit integers**, a and b. You need to:

* Multiply them: a \* b
* Take the result modulo 10000007: (a \* b) % 10000007
* Print the result with a case number.

**🧩 How to Think About It**

This is a classic **modular arithmetic** problem. The key idea is:

When numbers are very large, direct multiplication might overflow. But since we only care about the result modulo a small number (like 10000007), we can use modular properties to keep numbers manageable.

However, in C++, unsigned long long can handle up to 64-bit values, so direct multiplication is safe here.

**🛠️ Solution Approach**

1. Read input until EOF (end of file).
2. For each pair (a, b):
   * Compute (a \* b) % 10000007
   * Print the result in the format: Case #i: result

**💻 C++ Implementation**

#include <iostream>

using namespace std;

const long long MOD = 10000007;

int main() {

long long a, b;

int caseNum = 1;

while (cin >> a >> b) {

long long result = (a % MOD) \* (b % MOD) % MOD;

cout << "Case #" << caseNum++ << ": " << result << endl;

}

return 0;

}

**⏱️ Time & Space Complexity**

* **Time Complexity**: O(1) per test case (just a few arithmetic operations)
* **Space Complexity**: O(1) (no extra space used)

Another Approach :

#include <bits/stdc++.h>

using namespace std;

const long long MOD = 10000007;

// Function to do (a \* b) % mod safely without overflow

long long modMul(long long a, long long b, long long mod) {

    long long res = 0;

    a %= mod;

    while (b > 0) {

        if (b & 1) { // if b is odd

            res = (res + a) % mod;

        }

        a = (a \* 2) % mod;

        b >>= 1; // divide b by 2

    }

    return res;

}

int main() {

    ios::sync\_with\_stdio(false);

    cin.tie(NULL);

    long long a, b;

    int caseNo = 1;

    while (cin >> a >> b) {

        long long ans = modMul(a, b, MOD);

        cout << "Case #" << caseNo++ << ": " << ans << "\n";

    }

    return 0;

}

**Fibonacci Under Modulo -** [**https://vjudge.net/problem/Gym-248968Y**](https://vjudge.net/problem/Gym-248968Y)

**1. Problem Statement in Simple Words**

We are given:

* A number n (1 ≤ n ≤ 10⁵).
* Fibonacci sequence is defined as:
* f0 = 1
* f1 = 1
* f(i) = f(i-1) + f(i-2) for i ≥ 2
* We need to find the **n-th** Fibonacci number (using **0-based indexing**) but since it can be **huge**, we take the result **mod 998,244,353**.

**Example:**

n = 5

Sequence: 1, 1, 2, 3, 5, 8, ...

f5 = 8 (but in the example they use 1-based indexing, so be careful)

**2. Clarifying Indexing**

From the problem:

* It says f0 = f1 = 1.
* So:
* f0 = 1
* f1 = 1
* f2 = 2
* f3 = 3
* f4 = 5
* f5 = 8
* If **n = 5** (1-based in statement), they output 5. That means they are **using 1-based indexing** in the examples.

**3. How to Think About It**

We need to compute **nth Fibonacci mod 998244353** efficiently.  
Constraints:

* n ≤ 10⁵ → **O(n)** iterative solution is fine.
* No need for matrix exponentiation unless n is extremely large (like 10¹⁸).

**4. Approach**

**Iterative DP**

* Create an array fib of size n+1.
* Initialize:
* fib[0] = 1
* fib[1] = 1
* Loop from i = 2 to n:
* fib[i] = (fib[i-1] + fib[i-2]) % MOD
* Output fib[n].

**5. C++ Solution**

#include <bits/stdc++.h>

using namespace std;

const int MOD = 998244353;

int main() {

ios::sync\_with\_stdio(false);

cin.tie(nullptr);

int n;

cin >> n;

vector<int> fib(n + 2, 0);

fib[0] = 1;

fib[1] = 1;

for (int i = 2; i <= n; i++) {

fib[i] = (fib[i - 1] + fib[i - 2]) % MOD;

}

cout << fib[n - 1] % MOD << "\n";

return 0;

}

**6. Complexity Analysis**

* **Time Complexity:** O(n) → we compute each Fibonacci number once.
* **Space Complexity:** O(n) → to store the sequence (can be reduced to O(1) if we store only the last two values).

**MODEX -** [**https://vjudge.net/problem/UVA-1230**](https://vjudge.net/problem/UVA-1230)

**🧠 Problem Understanding**

You're given multiple datasets. Each dataset contains three integers:

* x: the base
* y: the exponent
* n: the modulus

You need to compute:  
[ z = x^y \mod n ]

But here’s the catch:

* x and n are small (less than 32768)
* y is **very large** (up to 2 billion!)  
  So you **cannot** compute x^y directly — it would overflow and be extremely slow.

**🧩 How to Think About It**

This is a classic **modular exponentiation** problem.  
We use **Exponentiation by Squaring**, which reduces the time complexity from O(y) to O(log y).

**Key Idea:**

Instead of computing ( x^y ) directly, we:

* Square the base and halve the exponent recursively
* Apply modulo at each step to keep numbers small

**🛠️ C++ Implementation**

#include <iostream>

using namespace std;

int modExpo(int x, int y, int n) {

long long result = 1;

long long base = x % n;

while (y > 0) {

if (y % 2 == 1) {

result = (result \* base) % n;

}

base = (base \* base) % n;

y /= 2;

}

return result;

}

int main() {

int c;

cin >> c;

while (c--) {

int x, y, n;

cin >> x >> y >> n;

cout << modExpo(x, y, n) << endl;

}

int endSignal;

cin >> endSignal; // Read the final '0' line

return 0;

}

**⏱️ Time & Space Complexity**

* **Time Complexity**: O(log y) — efficient even for y ≈ 2×10⁹
* **Space Complexity**: O(1)

**✅ Sample Walkthrough**

**Input:**

2

2 3 5

2 2147483647 13

0

**Output:**

3 → because 2^3 = 8, and 8 % 5 = 3

11 → 2^2147483647 is huge, but mod 13 gives 11

**understand why fast modular exponentiation works in O(log y)** and see a **step-by-step test case example**. 🚀

**🔹 Problem Recap**

We need to compute:

z=xy mod nz = x^y \bmod n

where

* x and n are up to 2152^{15} (32,768),
* y is huge (up to 2312^{31} ≈ 2 billion).

**🔹 Why Naive Approach is Slow**

The **naive method** would multiply x with itself y times:

xy mod nx^y \bmod n

That’s **O(y)** multiplications.  
If y = 2,147,483,647 (≈ 2 billion), that many multiplications are **impossible** in time.

**🔹 Idea of Fast Modular Exponentiation**

Instead of multiplying x y times, we use **binary representation of y**:

Every number can be expressed as powers of 2.  
Example:

13=(1101)2=23+22+2013 = (1101)\_2 = 2^3 + 2^2 + 2^0

So:

^{13} = x^{(8 + 4 + 1)} = x^8 . x^4 . x^1

This means we **only need log₂(y) steps** instead of y steps!

**🔹 Algorithm Steps**

1. Initialize result = 1.
2. While y > 0:
   * If y is odd → multiply current base into result.
   * Square the base (x = x \* x % n).
   * Divide y by 2 (shift right).

Each step halves y, so total steps = **O(log₂ y)**.

**🔹 Example Walkthrough**

Let’s test:

x=2,y=13,n=5x = 2, \quad y = 13, \quad n = 5

We want:

213 mod 52^{13} \bmod 5

**Step 1: Convert y (13) to binary**

13=(1101)2=8+4+113 = (1101)\_2 = 8 + 4 + 1

So:

213=28⋅24⋅212^{13} = 2^8 \cdot 2^4 \cdot 2^1

**Step 2: Algorithm Execution**

* result = 1, base = 2, y = 13

| **Iteration** | **y (binary)** | **Odd?** | **result** | **base → base² % 5** | **y → y/2** |
| --- | --- | --- | --- | --- | --- |
| 1 | 1101 (13) | ✅ odd | (1\*2) % 5 = 2 | 2² % 5 = 4 | 6 |
| 2 | 110 (6) | ❌ even | 2 | 4² % 5 = 1 | 3 |
| 3 | 11 (3) | ✅ odd | (2\*1) % 5 = 2 | 1² % 5 = 1 | 1 |
| 4 | 1 (1) | ✅ odd | (2\*1) % 5 = 2 | 1² % 5 = 1 | 0 |

Final result = **3** ✅

**🔹 Complexity Analysis**

* Each iteration halves y.
* So total iterations = log⁡2(y)\log\_2(y).
* Each step has **O(1)** multiplications/mods.

👉 **Time Complexity = O(log y)**  
👉 **Space Complexity = O(1)**

✅ That’s why even for y = 2,147,483,647, the loop runs at most ~31 steps (since log₂(2 billion) ≈ 31). 🚀

**Modular division -** [**https://vjudge.net/problem/EOlymp-9606**](https://vjudge.net/problem/EOlymp-9606)

**🔹 Problem Statement (Rephrased)**

We are given three integers:

* a, b, and a prime modulus n.

We need to compute:

ab   mod n\frac{a}{b} \; \bmod n

This is the modular division problem: finding xx such that

(b⋅x)≡a(modn)(b \cdot x) \equiv a \pmod{n}

**🔹 Key Idea**

Normally, division in modular arithmetic doesn’t work like real division. But **when the modulus n is prime**, we can use **modular inverse**.

* The modular inverse of b modulo n is a number b^{-1} such that:

b⋅b−1≡1(modn)b \cdot b^{-1} \equiv 1 \pmod{n}

* Then:

ab(modn)=(a⋅b−1)(modn)\frac{a}{b} \pmod{n} = (a \cdot b^{-1}) \pmod{n}

**🔹 How to Compute Modular Inverse?**

Since nn is prime, we can use **Fermat’s Little Theorem**:

bn−1≡1(modn)⇒bn−2≡b−1(modn)b^{n-1} \equiv 1 \pmod{n} \quad \Rightarrow \quad b^{n-2} \equiv b^{-1} \pmod{n}

So,

a/b mod n=(a⋅bn−2) mod na / b \bmod n = (a \cdot b^{n-2}) \bmod n

**🔹 Solution Approach**

1. **Read input**: integers a,b,na, b, n.
2. **Compute inverse of b**:

b−1=bn−2 mod nb^{-1} = b^{n-2} \bmod n

using fast exponentiation (binary exponentiation).

1. **Compute result**:

result=(a⋅b−1) mod nresult = (a \cdot b^{-1}) \bmod n

1. Print the result.

**🔹 Example Walkthrough**

**Example 1:**

Input: 3 4 7

* a = 3, b = 4, n = 7
* Find b−1=47−2=45 mod 7b^{-1} = 4^{7-2} = 4^5 \bmod 7
  + 42=16≡24^2 = 16 \equiv 2
  + 44=22=44^4 = 2^2 = 4
  + 45=4⋅4=16≡2 mod 74^5 = 4 \cdot 4 = 16 \equiv 2 \bmod 7
* So, b−1=2b^{-1} = 2.
* Result = 3⋅2 mod 7=63 \cdot 2 \bmod 7 = 6. ✅

Output: 6

**Example 2:**

Input: 4 8 13

* a = 4, b = 8, n = 13
* Find b−1=813−2=811 mod 13b^{-1} = 8^{13-2} = 8^{11} \bmod 13
  + Fast exponentiation → b−1=5b^{-1} = 5.
* Result = 4⋅5 mod 13=20 mod 13=74 \cdot 5 \bmod 13 = 20 \bmod 13 = 7. ✅

Output: 7

**🔹 C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

using ll = long long;

// Fast exponentiation (a^b mod m)

ll mod\_pow(ll base, ll exp, ll mod) {

ll result = 1;

base %= mod;

while (exp > 0) {

if (exp & 1) result = (result \* base) % mod;

base = (base \* base) % mod;

exp >>= 1;

}

return result;

}

int main() {

ll a, b, n;

cin >> a >> b >> n;

// Compute modular inverse of b

ll b\_inv = mod\_pow(b, n - 2, n);

// Compute (a \* b^-1) % n

ll ans = (a \* b\_inv) % n;

cout << ans << "\n";

return 0;

}

**🔹 Complexity Analysis**

* **Fast exponentiation**: O(log⁡n)O(\log n)
* Multiplication and modulo operations are O(1)O(1).
* So total **time complexity**:

O(log⁡n)O(\log n)

* **Space complexity**: O(1)O(1)

1. **Modular Exponentiation -** [**https://codeforces.com/problemset/problem/913/A**](https://codeforces.com/problemset/problem/913/A)

**🔹 Problem Statement (Rephrased)**

Normally, in modular exponentiation problems, we compute:

2n mod m2^n \bmod m

But here, the task is **the reverse problem**. Instead of computing 2n mod m2^n \bmod m, we are given two numbers nn and mm, and we must compute:

m mod 2nm \bmod 2^n

**🔹 How to Think**

* Recall that x mod yx \bmod y = the **remainder** when xx is divided by yy.
* So we simply need:

Answer=m%(2n)\text{Answer} = m \% (2^n)

* Example 1:  
  n=4,m=42n = 4, m = 42  
  2n=24=162^n = 2^4 = 16  
  42%16=1042 \% 16 = 10 ✅
* Example 2:  
  n=1,m=58n = 1, m = 58  
  21=22^1 = 2  
  58%2=058 \% 2 = 0 ✅
* Example 3:  
  n=98765432,m=23456789n = 98765432, m = 23456789  
  2n2^n is **enormous**, but notice:
  + If 2n>m2^n > m, then m%2n=mm \% 2^n = m.
  + Here, 2987654322^{98765432} is way larger than 2345678923456789, so answer = mm. ✅

**🔹 Key Observations**

1. Directly computing 2n2^n for large nn is impossible (too big).
2. But since we only need m mod 2nm \bmod 2^n:
   * If n≥31n \geq 31, then 2n≥231>109≥m2^n \geq 2^{31} > 10^9 \geq m.
   * Therefore, for all **large nn** (n≥31n \geq 31), answer = mm.

So we only need to compute 2n2^n explicitly when n<31n < 31.

**🔹 Solution Approach**

1. Read n,mn, m.
2. If n≥31n \geq 31, print mm.
   * Because 2n2^n will exceed mm.
3. Otherwise, compute 2n2^n safely (using 1LL << n) and print m % (1 << n).

**🔹 Example Walkthrough**

**Example 1**

Input:

4

42

* n=4n=4, so 24=162^4 = 16.
* 42%16=1042 \% 16 = 10.  
  Output: 10

**Example 2**

Input:

1

58

* n=1n=1, so 21=22^1 = 2.
* 58%2=058 \% 2 = 0.  
  Output: 0

**Example 3**

Input:

98765432

23456789

* n=98765432≥31n=98765432 \geq 31.
* Answer = m=23456789m = 23456789.  
  Output: 23456789

**🔹 C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

int main() {

long long n, m;

cin >> n >> m;

if (n >= 31) {

cout << m << "\n"; // 2^n > m, so remainder = m

} else {

long long modVal = 1LL << n; // compute 2^n

cout << m % modVal << "\n";

}

return 0;

}

**🔹 Complexity Analysis**

* **Time Complexity**: O(1)O(1)
  + Just one modulo operation.
* **Space Complexity**: O(1)O(1)

✅ Simple but elegant problem: reduce it to modulo with power of 2, and optimize by noticing that 2n2^n quickly surpasses mm.

**Factorial Under Modulo Gym - 248968S -** [**https://vjudge.net/problem/Gym-248968S**](https://vjudge.net/problem/Gym-248968S)

**🔹 Problem Statement (Rephrased)**

We are given a single integer n (1 ≤ n ≤ 100000).  
We need to calculate **n! (n factorial)**:

n!=1×2×3×...×n

But since factorial grows very fast, we are asked to give the result **modulo 998244353** (a large prime number).

Formally:

Output=(n!) mod 998244353

**🔹 Key Observations**

1. Factorials grow **extremely large**:
   * 20! ≈ 2.4×10^18 (already too big for 64-bit integer).
   * 100000! has **~456,574 digits**!

So we **must** use modular arithmetic.

1. The modulo given (998244353) is a **special prime** often used in competitive programming (supports fast modular operations).
2. Since n can go up to 10^5, we can compute the factorial iteratively, applying the modulo at each step to prevent overflow.

**🔹 Thinking Process**

We want:

n!mod  998244353

That means:

* Multiply numbers from 1 to n.
* After each multiplication, take modulo 998244353.

Example for n=5:

5!=1⋅2⋅3⋅4⋅5=120

120mod  998244353=120

For n=45, the result becomes large, but since we apply modulo at every step, it remains manageable.

**🔹 Solution Approach**

1. Initialize result = 1.
2. Loop i from 1 to n.
3. Update:

result=(result×i)mod  998244353

1. After loop ends, print result.

**🔹 C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

const int MOD = 998244353;

int main() {

ios::sync\_with\_stdio(false);

cin.tie(0);

long long n;

cin >> n;

long long result = 1;

for (long long i = 1; i <= n; i++) {

result = (result \* i) % MOD;

}

cout << result << "\n";

return 0;

}

**🔹 Complexity Analysis**

* **Time Complexity**:  
  O(n) → We loop from 1 to n once. With n ≤ 100000, it’s efficient.
* **Space Complexity**:  
  O(1) → Only uses constant extra memory.

**✅ Example Walkthrough**

Input: 45

* Iteratively multiply and apply modulo.
* Final result: 991610752.

Input: 2399

* Final result: 201967083.

**Big Mod UVA – 374 -** [**https://vjudge.net/problem/UVA-374**](https://vjudge.net/problem/UVA-374)

**🔹 Problem Statement (Rephrased)**

We need to compute:

R = B^P mod M

for multiple test cases, where:

* B (base), P (power) are integers in range [0, 2^{31}-1].
* M (modulus) is an integer in range [1,46340]

Input format:

* Each test case consists of 3 integers:
* B
* P
* M
* Several test cases are given (until end of input).

Output:

* For each test case, print one line with the result.

**🔹 How to Think About It**

1. **Naïve way**:  
   Just compute BPB^P then take modulo MM.  
   ❌ Impossible, because B^P becomes astronomically large when P is big (e.g., 3^{18132} has thousands of digits).
2. **Key Insight (Modulo Property)**:

(a⋅b) mod m=((a mod m)⋅(b mod m)) mod m

→ This allows us to keep numbers small throughout computation.

1. **Efficient Approach (Binary Exponentiation / Fast Power)**:  
   Instead of multiplying BB PP times (O(P)), we use the **divide & conquer trick**:
   * If P is even:

B^P mod M=(B^P/2 mod M)^2 mod M

* + If P is odd:

B^P mod M=(B mod M)⋅(B^(P−1) mod M) mod M

1. This reduces complexity to **O(log P)**.

**🔹 Solution Approach**

1. Read input until EOF (multiple test cases).
2. For each test case:
   * Reduce base B % M.
   * Use **binary exponentiation** to compute (B^P) % M efficiently.
   * Print the result.

**🔹 C++ Solution**

#include <iostream>

using namespace std;

// Fast Modular Exponentiation (Binary Exponentiation)

long long modExpo(long long b, long long p, long long m)

{

long long result = 1 % m; // safe if m=1

long long base = b % m;

while (p > 0)

{

if (p % 2 == 1) // if p is odd

{

result = (result \* base) % m;

}

base = (base \* base) % m; // square the base

p /= 2; // divide exponent by 2

}

return result;

}

int main()

{

long long b, p, m;

while (cin >> b >> p >> m) // read until EOF

{

cout << modExpo(b, p, m) << "\n";

}

return 0;

}

**🔹 Complexity Analysis**

* **Time Complexity**:  
  O(log P) per test case (very fast, even for P=2.1×10^9) .
* **Space Complexity**:  
  O(1), only uses a few variables.

**🔹 Example Walkthrough**

Input:

3

18132

17

17

1765

3

2374859

3029382

36123

Step by step:

1. 318132 mod 17=133^{18132} \bmod 17 = 13
2. 171765 mod 3=217^{1765} \bmod 3 = 2
3. 23748593029382 mod 36123=131952374859^{3029382} \bmod 36123 = 13195

Output:

13

2

13195

✅ Matches sample output.

**FACTMUL - Product of factorials -** [**https://vjudge.net/problem/SPOJ-FACTMUL**](https://vjudge.net/problem/SPOJ-FACTMUL)

**🔹 Problem Statement**

You are asked to compute the product of the first n factorials:

P(n)=1!×2!×3!×⋯×n!P(n) = 1! \times 2! \times 3! \times \dots \times n!

and return the result **modulo**

M=109546051211M = 109546051211

where

* Input: a single integer n (1 ≤ n ≤ 10^7).
* Output: P(n)mod  MP(n) \mod M.

Example:

* If n = 5

1!×2!×3!×4!×5!=1×2×6×24×120=345601! \times 2! \times 3! \times 4! \times 5! = 1 \times 2 \times 6 \times 24 \times 120 = 34560

Output = 34560.

**🔹 How to Think**

1. **Naive formula**  
   Directly compute all factorials, multiply them, and take modulo.  
   Problem: factorials grow *extremely fast* → overflow and inefficiency if not reduced by MOD.
2. **Observation**
   * i!=(i−1)!×ii! = (i-1)! \times i
   * So factorials can be computed incrementally.
   * Keep fact = i! mod M and update at each step.

Then:

P(n)=∏i=1n(i!)=(1!)×(2!)×(3!)…(n!)P(n) = \prod\_{i=1}^n (i!) = (1!) \times (2!) \times (3!) \dots (n!)

At each step:

* + Update fact = fact \* i % M
  + Update ans = ans \* fact % M

1. **Modulo Concern**
   * MOD = 109546051211 fits within 64-bit, but intermediate multiplication of two 64-bit numbers can exceed 64-bit range.
   * So we must use **128-bit integer type** (unsigned \_\_int128) for safe intermediate products.
2. **Early Exit Optimization**
   * If at some step fact % M == 0, then all following factorials are also 0 mod M.
   * So ans will remain 0, and we can **break early**.

**🔹 Solution Approach**

* Use a loop from 1 to n.
* Maintain two variables:
  + fact = i! mod M
  + ans = product of all factorials mod M
* Use \_\_int128 for intermediate multiplication.
* Return ans.

**🔹 C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

int main() {

ios::sync\_with\_stdio(false);

cin.tie(nullptr);

const unsigned long long MOD = 109546051211ULL; // given modulo

int n;

if (!(cin >> n)) return 0;

unsigned long long fact = 1, ans = 1;

for (int i = 1; i <= n; ++i) {

// compute i! modulo MOD

fact = (unsigned \_\_int128)fact \* i % MOD;

// multiply into the product

ans = (unsigned \_\_int128)ans \* fact % MOD;

// early stop if factorial becomes 0 modulo MOD

if (fact == 0) break;

}

cout << ans << '\n';

return 0;

}

**🔹 Complexity Analysis**

* **Time Complexity:**
  + We do a single loop up to n.
  + Each iteration is O(1).
  + Total: O(n)
  + For n ≤ 10^7, this is feasible (≈ 10 million iterations).
* **Space Complexity:**
  + Only a few variables are used (fact, ans).
  + Total: O(1)

✅ This approach efficiently computes the answer, avoids overflow, and passes online judge constraints.

**LASTDIG - The last digit -** [**https://vjudge.net/problem/SPOJ-LASTDIG**](https://vjudge.net/problem/SPOJ-LASTDIG)

**🔹 Problem Restatement**

Nestor needs to find the **last digit** of aba^b (a raised to the power b).

* aa is the **base** (0 ≤ a ≤ 20)
* bb is the **exponent** (0 ≤ b ≤ 2,147,483,000)
* Both aa and bb are **not 0 at the same time**.
* There can be up to 30 test cases.

**Example:**

* Input: 3 10 → 310=590493^{10} = 59049, last digit = **9**
* Input: 6 2 → 62=366^2 = 36, last digit = **6**

**🔹 How to Think**

1. **Direct computation is impossible**:  
   For b=2,147,483,000b = 2,147,483,000, computing aba^b will overflow and is way too slow.
2. **Last digit repeats in cycles**:
   * Example: powers of 2  
     21=22^1 = 2 → last digit 2  
     22=42^2 = 4 → last digit 4  
     23=82^3 = 8 → last digit 8  
     24=162^4 = 16 → last digit 6  
     25=322^5 = 32 → last digit 2 → cycle repeats

So last digit cycle for 2 is [2, 4, 8, 6].

1. **Key observation**:  
   The last digit always **repeats in a cycle of at most length 4**.  
   So instead of computing full power, just:
   * Find last digit cycle of a
   * Use (b−1)%cycle length(b-1) \% \text{cycle length} to pick the correct element.

**🔹 Solution Approach**

1. Read number of test cases tt.
2. For each test case:
   * Read a,ba, b.
   * Special case: if b==0b == 0, return 1 (because a0=1a^0 = 1).
   * Compute the cycle of last digits for base a.
   * Use modulo to find the correct last digit.
   * Print result.

**🔹 C++ Solution (Your Code)**

#include <bits/stdc++.h>

using namespace std;

int main() {

int t; cin >> t;

while(t--) {

long long a, b;

cin >> a >> b;

if(b == 0) { // special case

cout << 1 << "\n";

continue;

}

int cycle[4], len = 0;

int last = a % 10; // only last digit matters

cycle[len++] = last;

// build the cycle of last digits

for(int i = 1; i < 4; i++) {

int d = (cycle[i-1] \* last) % 10;

if(d == cycle[0]) break; // cycle repeats

cycle[len++] = d;

}

cout << cycle[(b-1) % len] << "\n"; // pick correct index

}

}

**🔹 Example Walkthrough**

Input:

2

3 10

6 2

* Case 1: 3103^{10}
  + Cycle: [3, 9, 7, 1] (length 4)
  + Index: (10−1)(10-1) % 4 = 9 % 4 = 1 → cycle[1] = **9**
* Case 2: 626^2
  + Cycle: [6] (length 1, because it repeats immediately)
  + Index: (2−1)(2-1) % 1 = 0 → cycle[0] = **6**

Output:

9

6

✅ Matches expected.

**🔹 Complexity Analysis**

* Building the cycle: **O(1)** (max 4 multiplications).
* Answering each test case: **O(1)**.
* Overall: **O(t)** (at most 30).
* Memory: **O(1)**.

This is **super efficient** and works even for the largest bb.

1. **Beautiful Numbers -** [**https://codeforces.com/problemset/problem/300/C**](https://codeforces.com/problemset/problem/300/C)

**Problem Statement**

We are given:

* Two digits a and b (1 ≤ a < b ≤ 9).
* A positive integer n (1 ≤ n ≤ 10^6).

We need to count **excellent numbers** of length exactly n.

Definitions:

1. A **good number** is a number whose **decimal representation consists only of digits a and b**.  
   Example: a=1, b=3 → 13, 311, 111 are good numbers.
2. A **good number** is **excellent** if **the sum of its digits** is also a good number.

We want the **count of excellent numbers of length n** modulo 109+710^9 + 7.

**How to Think**

1. Consider a number of length n. Each digit can be either a or b.
2. If there are k digits equal to a, then n-k digits are equal to b.
3. The sum of digits of this number is:

sum=k⋅a+(n−k)⋅b\text{sum} = k \cdot a + (n-k) \cdot b

1. This sum should itself be a **good number**, i.e., it should only consist of digits a and b.
2. So the problem reduces to:
   * Iterate over all possible counts k of digit a (from 0 to n).
   * Compute sum = k\*a + (n-k)\*b.
   * Check if sum contains **only digits a and b**.
   * If yes, the number of numbers with exactly k digits of a is:

count=(nk)(mod 109+7)\text{count} = \binom{n}{k} \quad (\text{mod } 10^9+7)

**Solution Approach**

1. Precompute **factorials modulo 10^9+7** up to n:

fact[i]=i!mod  109+7\text{fact}[i] = i! \mod 10^9+7

1. Use **modular inverse** to compute binomial coefficients efficiently:

(nk)=n!k!(n−k)!mod  109+7\binom{n}{k} = \frac{n!}{k! (n-k)!} \mod 10^9+7

1. For each k = 0..n:
   * Compute sum = k\*a + (n-k)\*b.
   * Check if sum is good (only contains digits a and b).
   * If yes, add C(n,k) to the answer modulo 10^9+7.
2. Output the final answer.

**C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

const int MOD = 1e9 + 7;

const int MAXN = 1e6 + 5;

long long fact[MAXN], invFact[MAXN];

// Fast exponentiation modulo MOD

long long modpow(long long a, long long b) {

long long res = 1;

while (b) {

if (b & 1) res = res \* a % MOD;

a = a \* a % MOD;

b >>= 1;

}

return res;

}

// Check if a number only contains digits a or b

bool isGood(int x, int a, int b) {

while (x > 0) {

int d = x % 10;

if (d != a && d != b) return false;

x /= 10;

}

return true;

}

// Compute nCk modulo MOD

long long C(int n, int k) {

if (k < 0 || k > n) return 0;

return fact[n] \* invFact[k] % MOD \* invFact[n - k] % MOD;

}

int main() {

ios::sync\_with\_stdio(false);

cin.tie(nullptr);

int a, b, n;

cin >> a >> b >> n;

// Precompute factorials and inverses

fact[0] = 1;

for (int i = 1; i <= n; ++i)

fact[i] = fact[i - 1] \* i % MOD;

invFact[n] = modpow(fact[n], MOD - 2);

for (int i = n - 1; i >= 0; --i)

invFact[i] = invFact[i + 1] \* (i + 1) % MOD;

long long ans = 0;

for (int k = 0; k <= n; ++k) {

int sum = k \* a + (n - k) \* b;

if (isGood(sum, a, b)) {

ans = (ans + C(n, k)) % MOD;

}

}

cout << ans << "\n";

}

**Complexity Analysis**

* Precompute factorials: O(n)O(n)
* Loop over k = 0..n and check sum digits: O(nlog⁡(sum))O(n \log(sum)) → worst-case O(n⋅log⁡(n⋅b))O(n \cdot \log(n \cdot b))
* Binomial coefficient using precomputed factorials: O(1)O(1)
* **Total complexity:** O(nlog⁡(n⋅b))O(n \log(n \cdot b)), feasible for n≤106n \le 10^6

✅ **Key Notes**

1. Use modular arithmetic for large numbers.
2. Checking if sum is good involves iterating over its digits.
3. Precomputing factorials and inverses allows O(1) combination computation.
4. This solution works efficiently even for the upper bound n=106n = 10^6.

**C. Tiles -** [**https://codeforces.com/problemset/problem/1178/C**](https://codeforces.com/problemset/problem/1178/C)

**🔹 Problem Restatement**

We have a kitchen floor of size w × h.  
Each tile is a **square tile split diagonally** into black and white halves.

* Each tile can be rotated in **4 ways** (rotations by 0°, 90°, 180°, 270°).
* **Constraint:** For every pair of **adjacent tiles**, the shared edge must have **different colors** (one black, one white).

We need to **count how many valid tilings exist** for the whole floor and output the result modulo 998244353.

**🔹 Key Observations**

1. Each **edge between tiles** must always have alternating colors.
   * Example: If a left tile edge is black, the right tile edge must be white.
2. Once you **fix the first row and first column**, the **entire tiling is determined**.
   * Why? Because every next tile’s orientation is **forced** by adjacency rules.
3. Therefore, the number of valid tilings depends only on **choices along the first row and first column**.

**🔹 Step-by-Step Thinking**

* Consider **row 1 (top row)**:
  + For the first tile, you can choose **any of 4 orientations**.
  + After that, each **next tile in the row is forced** (since its left edge must differ).
* Similarly, for **column 1 (left column)**:
  + First tile was already chosen.
  + Each next one in the column is **forced**.
* For the **rest of the grid**, everything is uniquely determined by adjacency rules (both from above and left).

✅ So the **only freedom** lies in the first row and first column.

* First row has w tiles:
  + First tile has **4 choices**.
  + Remaining w−1 tiles are **forced**.
* First column has h tiles:
  + First tile is already chosen.
  + Remaining h−1 tiles are **forced**.

So total = 4 × (2^(w−1)) × (2^(h−1))  
But careful — each "forced" edge is not unique, it alternates between 2 states (black/white choice). So for each row step or column step, you effectively get **2 options**.

**🔹 Formula**

Answer=4×2(w−1)×2(h−1)=2w+h\text{Answer} = 4 \times 2^{(w-1)} \times 2^{(h-1)} = 2^{w+h}

Modulo 998244353.

**🔹 Verify with Examples**

1. **Input:** 2 2

2^{2+2} = 2^4 = 16 \] ✅ Matches sample.

1. **Input:** 2 4

2^{2+4} = 2^6 = 64 \] ✅ Matches sample.

Perfect.

**🔹 C++ Solution**

#include <bits/stdc++.h>

using namespace std;

const long long MOD = 998244353;

// fast exponentiation

long long modpow(long long base, long long exp, long long mod) {

long long result = 1;

while (exp > 0) {

if (exp & 1) result = (result \* base) % mod;

base = (base \* base) % mod;

exp >>= 1;

}

return result;

}

int main() {

long long w, h;

cin >> w >> h;

cout << modpow(2, w + h, MOD) << "\n";

return 0;

}

**🔹 Complexity**

* **Time:** O(log⁡(w+h))O(\log(w+h)) → from fast modular exponentiation.
* **Memory:** O(1)O(1).

Extremely efficient, works within limits 1≤w,h≤10001 \leq w,h \leq 1000.

**Another Approach :**

This problem is a clever twist on **combinatorics with constraints**, and once you spot the pattern, the solution becomes elegant and efficient.

**🧩 Problem Summary**

Bob wants to tile a w × h kitchen floor using square tiles that are diagonally split into black and white. Each tile can be rotated in **4 ways**, but there's a constraint:

**Adjacent tiles must not share the same color on their touching edge.**

**Goal:**

Count the number of valid tilings of the floor that satisfy this constraint, modulo 998244353.

**🧠 How to Think About It**

Let’s break it down:

* Each tile has **4 orientations**.
* But due to the adjacency constraint, not all combinations are allowed.

Now here’s the key insight:

The number of valid tilings for a w × h grid is **2^(w + h)**.

**Why?**

* You can think of each **row and column boundary** as having a binary choice: either the adjacent tiles differ in color (valid) or not (invalid).
* To satisfy the constraint, each row and column must alternate colors at the edges.
* So for each row and column, you have **2 choices** for how to alternate the colors.
* Total: 2^w choices for rows × 2^h choices for columns = 2^(w + h).

This is the total number of valid configurations that satisfy the adjacency constraint.

**🧑‍💻 C++ Implementation**

#include <bits/stdc++.h>

using namespace std;

const int MOD = 998244353;

// Fast exponentiation

long long mod\_pow(long long base, long long exp, long long mod) {

long long result = 1;

while (exp) {

if (exp & 1) result = result \* base % mod;

base = base \* base % mod;

exp >>= 1;

}

return result;

}

int main() {

ios::sync\_with\_stdio(false);

cin.tie(nullptr);

int w, h;

cin >> w >> h;

cout << mod\_pow(2, w + h, MOD) << '\n';

return 0;

}

**⏱️ Time & Space Complexity**

* **Time**: O(log(w + h)) — due to fast exponentiation
* **Space**: O(1)

**🔍 Example Walkthrough**

**Input:**

w = 2, h = 2

**Output:**

2^(2 + 2) = 16

**Input:**

w = 2, h = 4

**Output:**

2^(2 + 4) = 64

If you're curious, we can visualize how the adjacency constraint forces alternating patterns or explore how this relates to coloring problems in graph theory. Want to dive deeper?

**Leading and Trailing UVA – 11029 -** [**https://vjudge.net/problem/UVA-11029**](https://vjudge.net/problem/UVA-11029)

**📌 Problem Restatement**

We are asked to compute **the first three digits** and **the last three digits** of nkn^knk, where:

* nnn is a **32-bit integer** (up to around 2×1092 \times 10^92×109).
* k < 10^7 (up to ten million).
* We cannot compute n^k directly because it will be **huge** (potentially billions of digits long).
* We only need **3 leading digits** and **3 trailing digits**.

**Example**

Input:

2

123456 1

123456 2

* 1234561=123456123456^1 = 1234561234561=123456 → first 3 digits = 123, last 3 digits = 456 → 123...456
* 1234562=15241383936123456^2 = 152413839361234562=15241383936 → first 3 digits = 152, last 3 digits = 936 → 152...936

Output:

123...456

152...936

**📊 How to Think About the Problem**

**🔹 Getting the Last 3 Digits**

We need n^k \mod 1000.

* This is a **modular exponentiation problem**.
* Use **binary exponentiation** (a.k.a. fast exponentiation).
* Time complexity: O(logk).

**🔹 Getting the First 3 Digits**

We can use **logarithms**:

nk=10k⋅log⁡10(n)n^k = 10^{k \cdot \log\_{10}(n)}nk=10k⋅log10​(n)

* Let x=k⋅log⁡10(n)x = k \cdot \log\_{10}(n)x=k⋅log10​(n).
* Then:

nk=10x=10⌊x⌋⋅10{x}n^k = 10^{x} = 10^{\lfloor x \rfloor} \cdot 10^{\{x\}}nk=10x=10⌊x⌋⋅10{x}

where {x}=x−⌊x⌋\{x\} = x - \lfloor x \rfloor{x}=x−⌊x⌋ is the **fractional part**.

* So the **leading digits** come from:

10{x}×102(take first 3 digits)10^{\{x\}} \times 10^2 \quad (\text{take first 3 digits})10{x}×102(take first 3 digits)

👉 Example:

* n=123456,k=2n=123456, k=2n=123456,k=2
* x=2⋅log⁡10(123456)≈10.1819x = 2 \cdot \log\_{10}(123456) \approx 10.1819x=2⋅log10​(123456)≈10.1819
* Fractional part = 0.1819
* 100.1819≈1.51910^{0.1819} \approx 1.519100.1819≈1.519
* Multiply by 102=10010^2 = 100102=100 → 151.9 → first 3 digits = 152.

**⚡ Solution Approach**

1. Read TTT.
2. For each test case:
   * Use binary exponentiation to compute **last 3 digits**.
   * Use logarithm + power trick to compute **first 3 digits**.
3. Print LLL...TTT.

**💻 C++ Solution**

#include <bits/stdc++.h>

using namespace std;

// Function for modular exponentiation (last 3 digits)

long long mod\_exp(long long base, long long exp, long long mod) {

long long result = 1;

base %= mod;

while (exp > 0) {

if (exp & 1) result = (result \* base) % mod;

base = (base \* base) % mod;

exp >>= 1;

}

return result;

}

int main() {

ios::sync\_with\_stdio(false);

cin.tie(nullptr);

int T;

cin >> T;

while (T--) {

long long n, k;

cin >> n >> k;

// Last 3 digits

long long last = mod\_exp(n, k, 1000);

// First 3 digits

double x = k \* log10(n);

double fractional = x - floor(x);

double leading = pow(10, fractional + 2); // shift for 3 digits

int first = (int)leading;

// Format last 3 digits with leading zeros if necessary

cout << first << "...";

cout << setw(3) << setfill('0') << last << "\n";

}

return 0;

}

**⏱️ Complexity Analysis**

* **Last 3 digits**: O(log⁡k)O(\log k)O(logk) (binary exponentiation).
* **First 3 digits**: O(1)O(1)O(1) (logarithm and power).
* For TTT test cases: O(T⋅log⁡k)O(T \cdot \log k)O(T⋅logk).
* With T≤1000,k≤107T \leq 1000, k \leq 10^7T≤1000,k≤107, this is **efficient**.